Mathematics College and Career Readiness Standards for Adult Education: Exploring the Instructional Shifts *Focus* and *Coherence*

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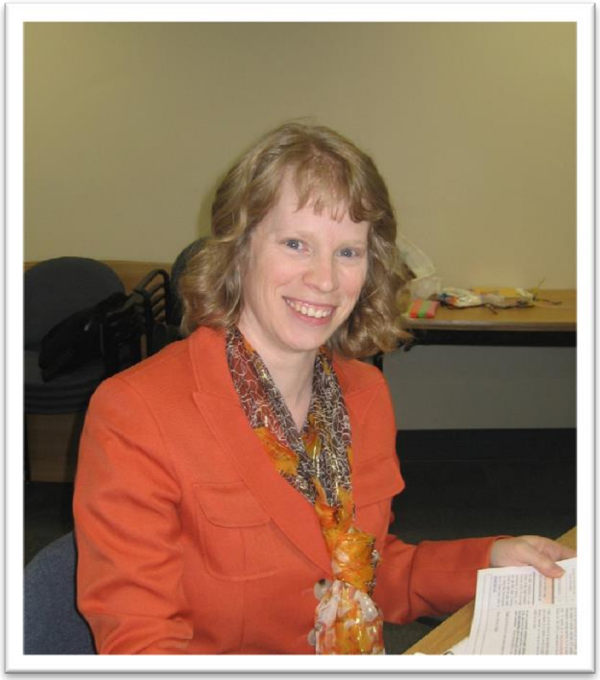

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Getting oriented

- The Common Core and College and Career Readiness Standards
- Referencing the document
- There are two webinars on the math instructional shifts

Why focus on the instructional shifts?

- Cross-state (NELRC) interest in addressing CCR standards, which are relevant to all assessments
- Instructional shifts are expectations of teachers (vs. standards as expectations of students)
- Instructional shifts are valuable for addressing the gap between current HS completion levels and college readiness, and for promoting more strategic, critical thinking.

Objectives:

- Introduce participants to the three CCR Standards Shifts
- Explore how these two shifts (focus and coherence) influence the structure of the CCR Standards and the order that standards should be taught
- Consider how each of these two shifts plays out in the adult education math classroom

The Mathematics Instructional Shifts

- **Focus**
- **Coherence**
- **Rigor**

Design and Organization of Math Standards

Level

Domain

Standard Statement

Cluster of Standards

LEVEL B (2-3)

Number and Operations: Base Ten

Understand place value.

Understand that the three digits of a three-digit number represent amounts of hundreds, tens, and ones; e.g., 706 equals 7 hundreds, 0 tens, and 6 ones. Understand the following as special cases:

a. 100 can be thought of as a bundle of ten tens — called a "hundred."

b. The numbers 100, 200, 300, 400, 500, 600, 700, 800, 900 refer to one, two, three, four, five, six, seven, eight, or nine hundreds (and 0 tens and 0 ones). (2.NBT.1)

Count within 1000; skip-count by 5s, 10s, and 100s. (2.NBT.2)

Read and write numbers to 1000 using base-ten numerals, number names, and expanded fo (2.NBT.3)

Compare two three-digit numbers based on meanings of the hundreds, tens, and ones and < symbols to record the results of comparisons. (2.NBT.4)

Use place value understanding and properties of operations to add and subtract.

Add up to four two-digit numbers using strategies based on place value and properties of operations. (2.NBT.6)

Add and subtract within 1000, using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method. Understand that in adding or subtracting three-digit numbers, one adds or subtracts hundreds and hundreds, tens and tens, ones and ones; and sometimes it is necessary to compose or decompose tens or hundreds. (2.NBT.7)

Mentally add 10 or 100 to a given number 100–900, and mentally subtract 10 or 100 from a given number 100–900. (2.NBT.8)

Design and Organization of Math Standards

- Levels A – D separated into a range of domains (such as The Number System, Operations and Algebraic Thinking, Functions, Geometry, Measurement and Data, Statistics and Probability)
- Level E organized by conceptual categories (Number and Quantity, Algebra, Functions, Modeling, Geometry, and Statistics and Probability)
- Standards are bundled into five grade-level groups: A (K-1), B(2-3), C (4-5, 6), D(6, 7-8), E (high school)
- Two central parts: Standards for Mathematical Practice and Standards for Mathematical Content, plus key shifts

The Mathematics Instructional Shifts

- **Focus**
- **Coherence**
- **Rigor**

Focus

Focus requires that we significantly narrow the scope of content in each grade and deepen how time and energy is spent on major topics in the classroom.

Why the CCSS and CCRS?

The single most important flaw in United States mathematics instruction is that the curriculum is "*a mile wide and an inch deep*."

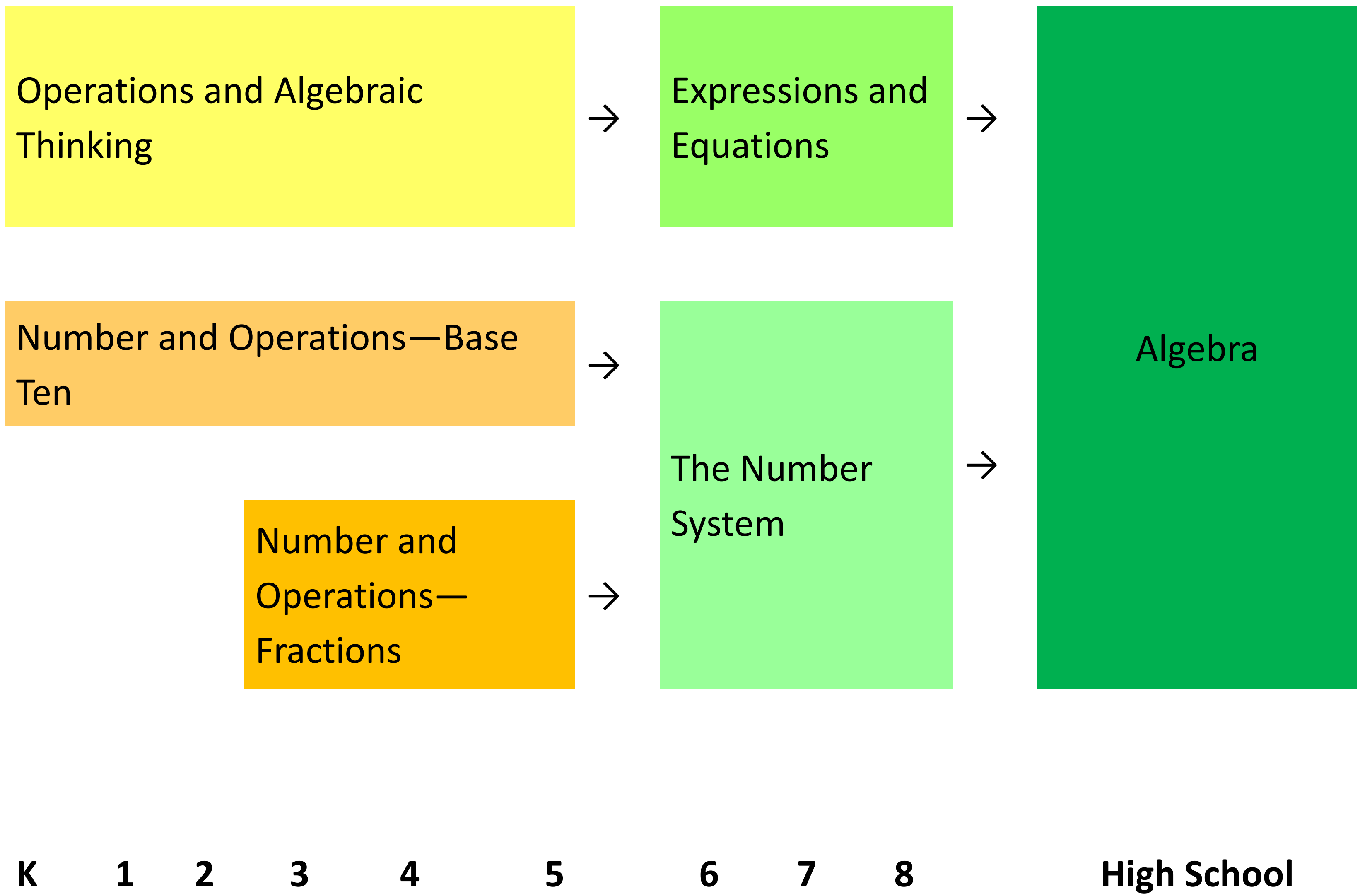
The standards are meant to be a blueprint for math instruction that is more focused and coherent.

—Daro, McCallum, and Zimba, 2012 (from the CCSS Appendix)

Domains in the CCR Standards

Domains	A	B	C	D	E
Number and Operations: Base Ten					
Number and Operations: Fractions					
The Number System					
Number and Quantity: The Real Number System and Quantities					
Ratios and Proportional Relationships					
Operations and Algebraic Thinking					
Expressions and Equations					
Functions					
Algebra					
Geometry					
Measurement and Data					
Statistics and Probability					

Focusing Attention Within Number and Operations



www.achievethecore.org

Focus: Instructional Implications

- Use the "Power of the Eraser."
- Many lessons in textbook curricular programs will need to be eliminated.
- Lessons will need to be identified or created.

Focus in Adult Education

Focus requires that we significantly narrow the scope of content at each level and deepen how time and energy is spent on major topics in the classroom.

We often hear the following statement, "I don't have time to deepen the content. Our students are with us for only a short period of time."

What is your response to that statement?

Please respond to the following two questions in the poll:

Which of the following do you teach your students [check as many as apply]?

Which of the following do you directly teach your students [check as many as apply]?

Teaching procedures without understanding

Hmmm. 5 goes into 9 one time. 1 x 5 is 5. Subtract 4 from 9.

```
    1
5 )92
   5
  4
```

Now what? Hmmm. 5 doesn't go into 4 so I have to bring down the 2. Now I have to think how many 5s go into 42.

```
   18
5 )92
   5
   42
   40
    2
```

Hmmm. 5 goes into 90 many more times than 1. I think 5 goes into 90 at least 9 times since 5 x 9 is 45.

$$\begin{array}{r l}
5\,\overline{)92} & \\
\underline{45} & \quad \mathbf{9} \\
47 &
\end{array}$$

Hmmm, I can see that there are another 9 sets of 5 in 47.

$$\begin{array}{r l}
5\,\overline{)92} & \\
\underline{45} & \quad \mathbf{9} \\
47 & \qquad + \\
\underline{45} & \quad \mathbf{9} \\
2 &
\end{array}$$

Hmmm. I can’t take 8 from 4, so I have to borrow.

$$\begin{array}{r} 94 \\ -\quad 38 \\ \hline \end{array}$$

Hmmm, I change the 9 to 8 and the 4 to 14. I think that’s the process.

$$\begin{array}{r} {\scriptstyle 8\ 14} \\ \cancel{9}4 \\ -\quad 38 \\ \hline \end{array}$$

Hmmm. If I use some of the properties I know, I won't have to deal with that subtraction algorithm.

$$\begin{array}{r} 94 \\ -\ \ 38 \\ \hline \end{array}$$

I actually have several options, but the two most obvious are either to change the first number so that I don't have to borrow, or I can change the second number. I could make 94 into 98 which would mean adding 4, or I could add 2 to 38 to make it 40.

What allows me to play around with any numbers are properties. In this case, I could use the additive inverse.

$$\begin{array}{r} 94 \\ -\ 38 \\ \hline \end{array}$$

$94 \ \color{red}{+4} = 98$

$-38 \ \color{red}{+ -4} = 42$

$94 \ \color{red}{+ 2} = 96$

$-38 \ \color{red}{+ -2} = 40$

Please respond to the following two questions in the poll:

Which of the following do you most often spend the most time on? [check one only]

Which of the following do you most often have to reteach? [check one only]

Why do you think you have to spend so much time teaching and reteaching certain topics?

The Mathematics Instructional Shifts

- Focus
- Coherence
- Rigor

Coherence

Coherence is about making math make sense. Mathematics is not a list of disconnected tricks or mnemonics. It is an elegant subject in which powerful knowledge results from reasoning with a small number of principles such as place value and properties of operations. The standards define progressions of learning that leverage these principles as they build knowledge over the grades.

Coherence

Mathematics is not a list of disconnected tricks or mnemonics. It is an elegant subject in which powerful knowledge results from reasoning with a small number of principles such as place value and properties of operations.

How does this description about what "mathematics" is corroborate or contradict your beliefs about teaching?

Coherence: Instructional Implications

- Connect the learning across levels.
- Links from one level to the next allow students to progress in their mathematical education.
- Understand that each standard is not a new event but an extension of previous learning.

Coherence: Building on previous understanding

- Extend understanding of fraction equivalence and ordering [4.NF.1]
- Build fractions from unit fractions by applying and extending previous understanding of operations on whole numbers. [4.NF.3]
- Apply and extend previous understandings of multiplication to multiply a fraction by a whole number. [4.NF.4]
- Apply and extend previous understandings of multiplication and division to divide fractions. [5.NF.4]
- Apply and extend previous understandings of multiplication and division to divide fractions by fractions. [6.NS.1]

Coherence - Building on Multiplication and Division

Apply and extend previous understanding of multiplication and division to multiply and divide fractions. [5.NF]

So, what do you know about multiplication of whole numbers that you can apply to multiplication of fractions? Don't fractions get smaller when you multiply them??

Coherence – Building on Properties

- Apply and extend previous understanding of arithmetic to algebraic expressions. [6.EE]
- Apply the properties of operations to generate equivalent expressions. For example, apply the distributive property to the expression 3(2 + x) to produce the equivalent expression 6 + 3x; apply the distributive property to the expression 24x + 18y to produce the equivalent expression 6(4x + 3y); apply properties of operations to y + y + y to produce the equivalent expression 3y. [6.EE.2c]

When should we introduce properties of operations?

Please respond to the following questions in the poll:

When do you typically introduce the associative property?

When do you typically introduce the inverse property of addition?

When do you typically introduce the distributive property?

Coherence: Alignment in Context

One of several staircases to algebra and properties.

Operations and Algebraic Thinking

[Level A]

Apply properties of operations as strategies to add and subtract. *Examples: If 8 + 3 = 11 is known, then 3 + 8 = 11 is also known. (Commutative property of addition.) To add 2 + 6 + 4, the second two numbers can be added to make a ten, so 2 + 6 + 4 = 2 + 10 = 12. (Associative property of addition.)* **(1.OA.3)**

Operations and Algebraic Thinking

[Level B]

Apply properties of operations as strategies to multiply and divide. *Examples: If 6 x 4 = 24 is known, then 4 x 6 = 24 is also known. (Commutative property of multiplication.) 3 x5 x 2 can be found by 3 x 5 = 15 ,then 15 x 2 = 30. (Associative property of multiplication.) Knowing that 8 x 5 = 40 and 8 x 2 = 16, one can find 8 x 7 as 8 x (5 + 2) = (8 x 5) + (8 x 2) = 40 + 16. (Distributive property.)* **(3.OA.5)**

Expressions and Equations

[Level C]

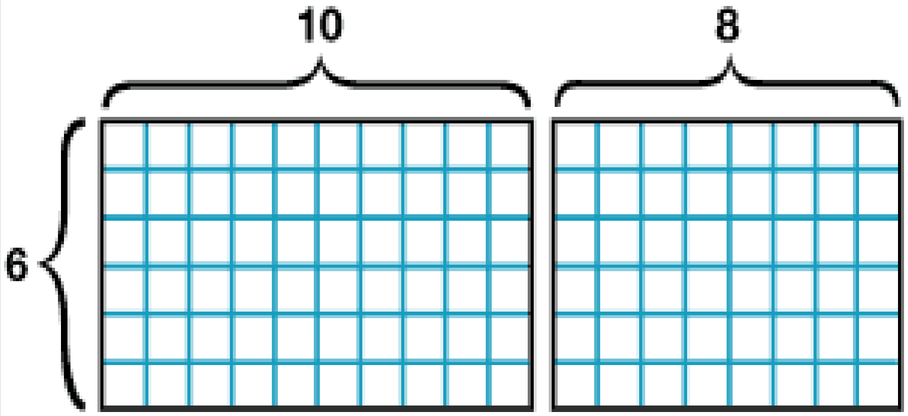
Apply the properties of operations to generate equivalent expressions. *For example, apply the distributive property to the expression 3 x (2 + x) to produce the equivalent expression 6 + 3x; apply the distributive property to the expression 24x + 18y to produce the equivalent expression 6(4x + 3y); apply properties of operations to y + y + y to produce the equivalent expression 3y.* **(6.EE.3)**

Expressions and Equations

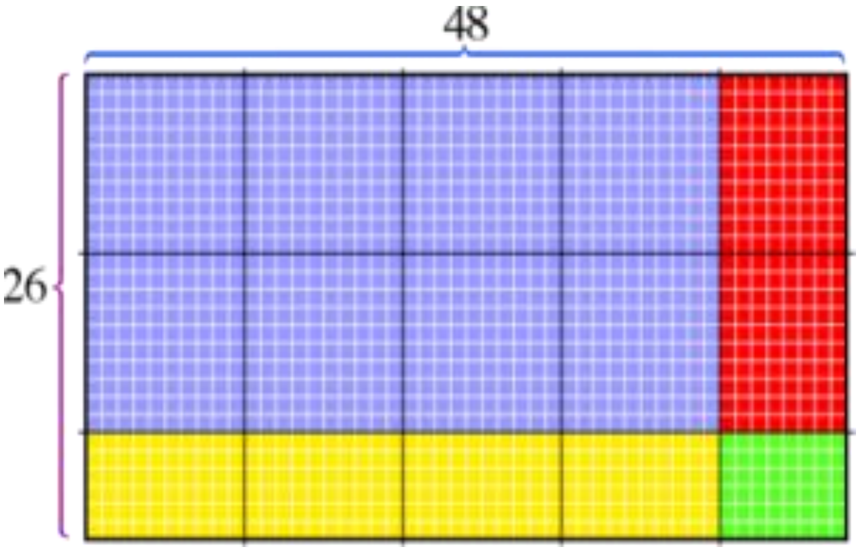
[Level D]

Apply properties of operations as strategies to add, subtract, factor, and expand linear expressions with rational coefficients. **(7.EE.1)**

Area Model of Multiplication



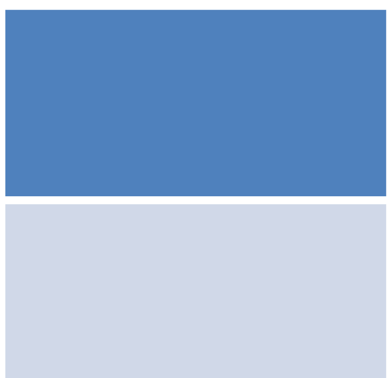
$$6 \times (10 + 8) = (6 \times 10) + (6 \times 8)$$
$$= 60 + 48$$
$$= 108$$

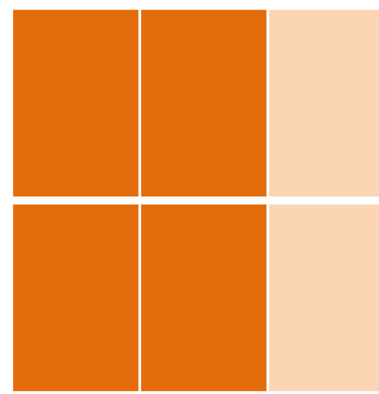


Let’s take a look at the area model using fractions:
1/2 x 2/3

Here is the whole.

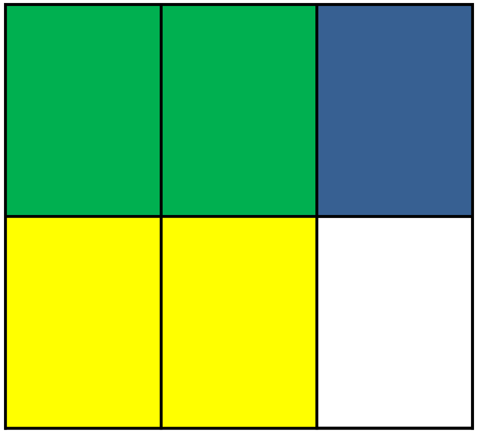
Here is the whole cut into halves; ½ is shaded





Here is the whole cut into thirds; 2/3 is shaded

1/2 x 2/3



1/2 x 2/3 = 2/6

Now let's take a look at the area model with a little algebra:

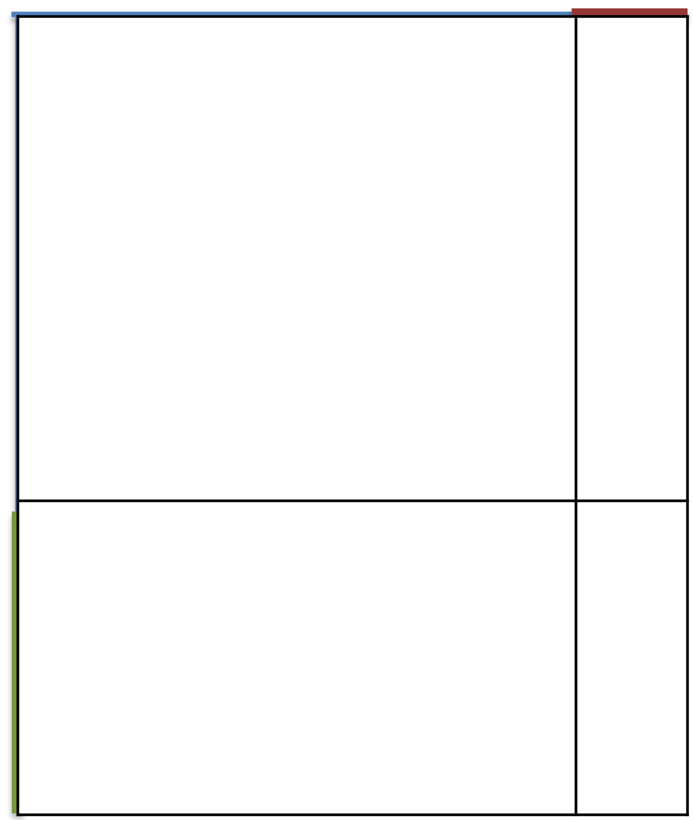
$(x + 2)(x + 5)$

Imagine this to be x.

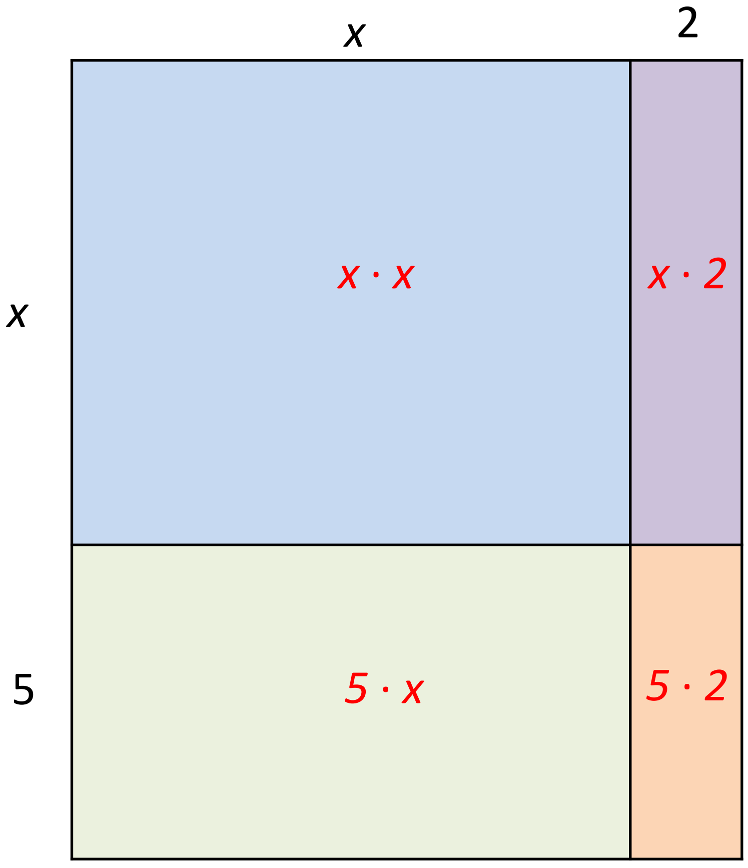
And imagine that this is 2

Here is another x

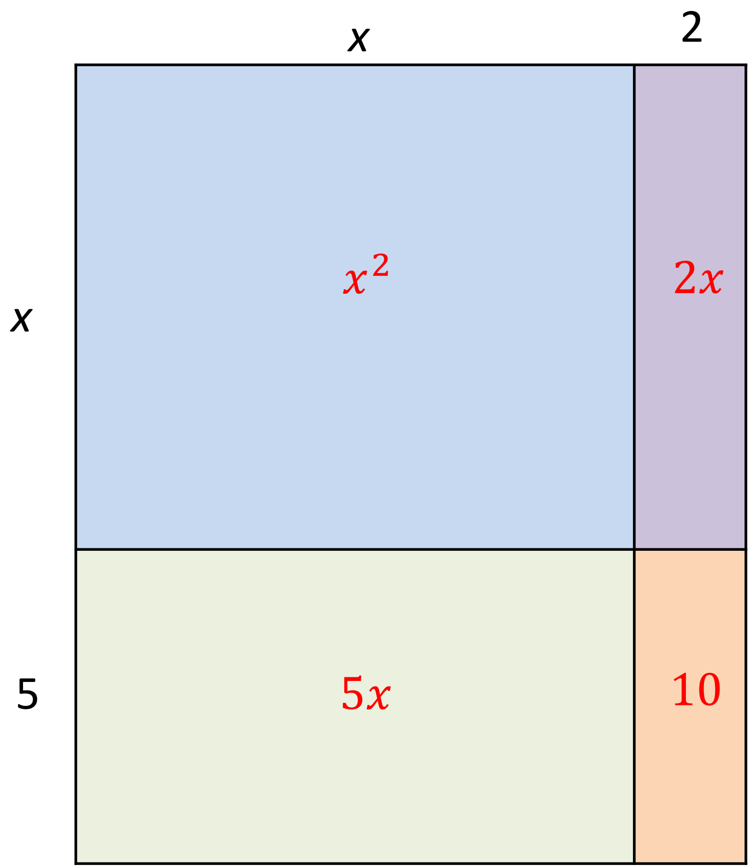
And here is 5 which we'll add to the x



$(x + 2)(x + 5)$



$(x + 2)(x + 5)$



Now add all the areas together:

$x^2 + 2x + 5x + 10$

…and simplify:

$x^2 + 7x + 10$

Focus and Coherence in Adult Education Classrooms

Focus requires that we significantly narrow the scope of content at each level and deepen how time and energy is spent on major topics in the classroom.

Coherence is about making math make sense. Mathematics is not a list of disconnected tricks or mnemonics. It is an elegant subject . . . with a small number of principles such as place value and properties of operations.

What do coherence and focus in an adult education classroom mean to you? How might these shifts influence what and how you teach?

Thank you!

- To our presenters: Donna, Ronnie, and Connie for summarizing and illustrating these concepts for us
- To you for being in this conversation!

Reminders

- Next math shifts webinar is on Feb. 25, 3:30-5:00 (you don’t have to re-register)
- The webinars will be recorded and archived at www.nelrc.org
- Please respond to the short evaluation survey to follow